Light Transport in Participating Media

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1 Introduction

Image synthesis of natural scenes is an important and challenging problem. The appearance of natural phenomena has always been intriguing. The fascination is evident from the vast amount of depictions of natural phenomena created, ranging from early cave paintings to impressionistic masterworks and photography.

The visual simulation of natural scenes has many practical applications. Many industries, from entertainment to architectural design, are using computer generated imagery of outdoor terrain scenes for their purposes. Therefore, it is important to design convincing visual simulation for natural scenes. For example, in flight simulators, it is critical that clouds and terrain be carefully depicted because they serve as vital visual cues to a pilot. On the other hand, the entertainment industry requires control in order to create desired effects. Therefore, the simulation of natural phenomena is subjected to two constraints: visual consistency and user control.

While scientists need to predict and simulate the behavior of certain phenomena, artists want to choreograph it in order to create a desired mood or effect. Movies require special effects not commonly found in nature or occurring very rarely. Even if the real phenomena can be found, controlling it can be difficult and cumbersome task. This makes modeling and rendering even more challenging problem. While purely physically-based simulation would yield a predictable result, the number of parameters controlling the simulation is unwieldy and unintuitive for non-expert users.

Significant progress has been made in the last decade in understanding how to generate realistic renderings of indoor scenes. The general approach is to analyze the physics of light transport in such environments and then to embody approximations to the physics in computational algorithms. Correct modeling of illumination and material properties is vital. It is now known that a sense of realism depends critically on accounting for shadows, secondary illumination, and non-uniform reflectance functions. Accurately approximating the effect of these properties involves great computational expense. As a result, methods for rendering realistic imagery almost always exploit assumptions about the nature of the geometric structure and illumination and materials properties likely to be encountered. Most of these assumptions derive from a presumption of indoor environments.

Natural scenes, especially outdoor scenes, present very different computational characteristics than indoor scenes. While the physics is the same, geometry, illumination, and reflectance properties are all distinctly different. Many of the techniques developed to support realistic rendering of indoor scenes require substantial modifications for natural, outdoor environments. The most difficult computational problem to overcome is the need to be able to aggregate the effects of micro-structures into large enough units that they can be rendered effectively, while at the same time preserving key aspects of visual appearance. This problem exists across a wide range of scales, ranging from foliage, in which a collection of individual leaves generates a collective appearance that is quite different than that of the constituent members, to distant landmarks, where detail must be suppressed without removing those properties that make landmarks distinctive and thus useful.

1.1 Why the Outdoors?

There are many reasons why study appearance, illumination and light transport in natural environments:

- **Great beauty**
- **Geometric, material and illumination richness and complexity**
- **Many modes of light transport** Light transport in natural environments is extremely complex as it varies from simple modes that can be described

by traditional shading paradigms to very complicated modes that require correct physical description and full simulation.

- **Challenging to many classic algorithms** Due to complex interactions between geometry, illumination and material properties, many "traditional" algorithms cannot handle natural environments adequately. Many existing models merely apply methods devised for "indoor" environments to natural scenes. Most often visual appearance of rendered natural scenes is not adequate. Furthermore, computational demands of natural scenes are overwhelming and "smarter" methods are needed to cope with complexity and computational demands.

- **There has been lots of research focusing on man-made materials and indoor environments, but not that much on natural and outdoor environments.**

Figure 1 shows the richness of illumination, geometric complexity and reflectances.

2 Motivation

Renderings of natural outdoor scenes have had a cartoon-like quality that significantly distracts from a sense of realism. Partially, this is due to computational and source data constraints that limit the geometric complexity of scenes to be rendered. On the other hand, illumination variety, complex reflectance functions and complex and not fully understood interactions between geometric complexity, illumination and material properties has also severely limited realism of natural scenes.
Why is the quality of computer generated images of the natural outdoor scenes inadequate?

- Source data and current computational constraints limit the geometric complexity of the environment.

- Illumination plays an equally important role in creating a sense of realism.

- We do not yet fully understand interactions between geometry, illumination and material properties.

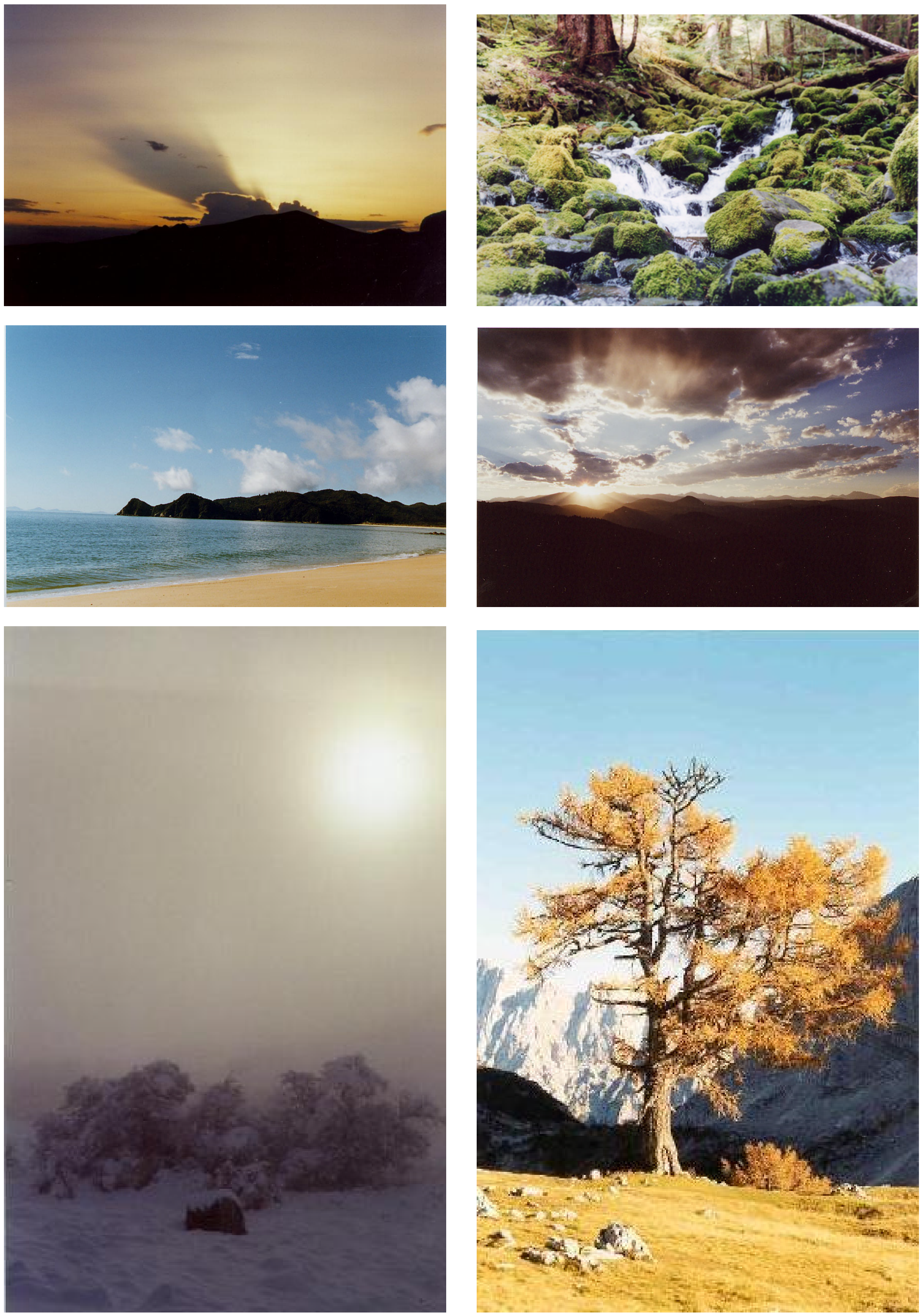

Figure 1: *Richness and variety of illumination, geometric complexity and reflectances found in natural scenes.*

Deussen *et. al.* demonstrated that improving geometric complexity of the scene also improves the perceived realism of the scene without any fancy illumination or material properties [15]. There has been a lot of research and progress made in understanding light transport in indoor environments and then approximating the physics in computational algorithms.

3 Problems

3.1 Illumination and Appearance

The human observer routinely has to deal with objects that are far from Lambertian. Many objects ubiquitous in the daily environment strongly deviate from Lambertian or Phong. Natural materials such as biological objects (leaves, skin), food (milk, fruits), or inorganic objects (sky, water, snow, clouds, weathered materials, rocks) exhibit significant subsurface or volumetric light transport. Light transport in arbitrary scattering media is very important for realistic depiction of materials [37] and scenes [57]. Many applications ranging from special effects to flight simulators and architectural design rely on subtle lighting effects and cues that often cannot be described by simplifying the light transport equation and without including effects of multiple scattering and global illumination within a scattering medium [10, 11].

For example, translucent material frequently occur in nature. The effects of translucency occur at quite different scale. While in some cases (e.g. cheese) the conventional shading paradigm may work reasonably well on the scale of the object, translucency becomes apparent in the appearance of small cracks and sharp edges. In other cases (clouds, smoke) the shading paradigm is fully inadequate.

Illumination and material appearance are at the heart of computer graphics. At the lowest level they are controlled by complex scattering events that are computationally expensive to model, hard to understand and cumbersome to control in practical applications. This is especially true for illumination and appearance in the natural outdoor environments. [11] shows that "many common observations cannot be explained by single-scattering arguments: the variation of brightness and color of the sky; the brightness of clouds; the whiteness of a glass of milk; the appearance of distant objects; the blueness of light transmitted in snow and other natural ice bodies; the darkening of sand upon wetting." While scattering events determine the illumination and appearance, it is very cumbersome to illumination and appearance using pure physical quantities such as particle density distributions, scattering coefficents, phase functions, etc. Most often these quantities are not known and are very non-intuitive for non-expert users. While numerical methods such as Monte Carlo methods ultimately produce the correct light distribution in an environment (or material), computational cost involved in accounting for all scattering events is prohibitive.

Accurate computation of light transport is therefore very complex, computationally expensive and sometimes hard to control and understand for an inexperienced user. For image synthesis purposes, approximations with intuitive parameters may often be enough to capture the appearance of almost any material.

3.2 Global Illumination

For many years, the goal of realistic image synthesis has been to simulate reality. This goal has driven the field to create a large number of algorithms to solve the light transport equation. Smits [63] argues that "none of which [the algorithms] are or will be practical for most application. In general, this is caused by ignoring the applications when designing algorithms. The opposite effect also takes place. An algorithm that focuses on a particular application tends to fail dramatically in other application areas, and tends to be criticized for this. A better understanding of the problem space should allow us to create better algorithms and give us better standards by which to evaluate algorithms."

While Deussen *et. al.* demonstrated that the realism of natural scenes can be greatly improved by increasing geometric complexity [15] using only local illumination, the importance of global illumination for realistic image synthesis has been demonstrated in recent experiments [24, 41, 67]. The presence of shadows, specular reflections, caustics, and diffuse interreflection provide important cues to the human visual system and help determine relationships between objects. Ward [70] posed a question of "how correct do these details need to be in order to be convincing?"

While Smits [63] pointed out that most of the current global illumination algorithms are not practical for real scenes, this is even more true for natural scenes that tend to be much more complex in terms of geometric complexity, materials, illumination and modes of light transport. Most existing algorithms are therefore impractical for solving global illumination in natural scenes. Furthermore, there have been recent trends in replacing complex geometry with point primitives [14, 66]. This addresses the geometric complexity issue and makes it managable. However, this introduces another problem of global or semi-global light interactions between these point primitives. None of the existing algorithms is appropriate for this new representation. While some attempts have been made to make global illumination computation more manageable from the systems point of view [54], it is clear that more effective algorithms are needed if the realism of natural scenes is to be improved.

4 Scattering and Light Transport

Interaction of light with particles is a fundamental physical phenomena that helps explaining the appearance of surfaces and arbitrary volumetric materials and participating media. *Scattering* is a process by which a particle or surface interacts with light. Scattering has a number of variations depending on the size of interacting particles. If interacting particles are much smaller than the wavelength λ of the incident light, the process is called *Rayleigh scattering*. Molecules found in the atmosphere fall into this category and blue sky is consequence of Rayleigh scattering. On the other hand, *Mie scattering* models scattering by particles that are roughly the same size as the wavelength λ. Water vapor, fumes, dust are the main scatters in the Earth's atmosphere. This type of scattering is responsible for spectacular red/orange appearances of the sky in the evenings, especially if there has been a forest fire, or a volcanic eruption. *Non-selective scattering* occurs when the particles are much larger than the incident radiation. This type of scattering is not wavelength dependent and is the primary cause of haze. Scattering process in which light undergoes scattering only once is called *single scattering*. Many common phenomena such as the appearance of clouds, brightness and color variations of the sky, aerial perspective cannot be explained by single scattering [11]. *Multiple scattering* is a scattering process in which light undergoes more than one interaction with particles. Figure 2 illustrates scattering in participating media. A basic understanding of scattering is required for understanding of appearance of natural phenomena and illumination. In the following sections we describe scattering process in more detail.

Optical Properties

In an arbitrary medium, the underlying optical properties at location $\mathbf{x}$ in space depend on bulk material properties such as *density* $\rho(\mathbf{x})$, *temperature* $T(\mathbf{x})$, and the particle absorption and scattering cross-sections, σ_a and σ_s. Optical properties of the medium are then described in terms of the *scattering coefficient*

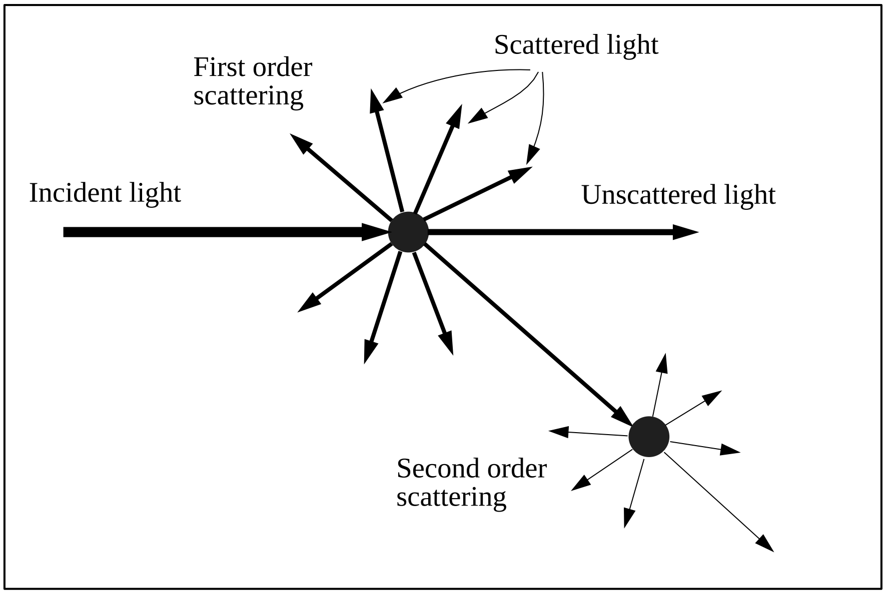

Figure 2: Scattering. (Figure courtesy of AJ Preetham)

$b(\mathbf{x}) = \sigma_s \rho(\mathbf{x})$, the *absorption coefficient* $a(\mathbf{x}) = \sigma_a \rho(\mathbf{x})$, the *extinction coefficient* $c(\mathbf{x}) = a(\mathbf{x}) + b(\mathbf{x})$, and the *phase function* $P(\mathbf{x}, \vec{\omega}, \vec{\omega}')$. Absorption and scattering coefficients are typically measured in inverse units of length (mm^{-1} or m^{-1}). Reciprocal of these coefficients is the average distance that light will travel before being absorbed or scattered. *Single scattering albedo* $W = \frac{b}{a+b}$ is the ratio of scattering to the sum of scattering and absorption. It is the percentage of all scattering events that are not absorption events. If $W = 1$, there is no absorption in the medium. Conversely, if $W = 0$, there is no scattering and the light is only attenuated due to absorption. Another important optical property is *diffuse attenuation* $K(\mathbf{x})$ which can be written in terms of the extinction coefficient (*beam extinction*): $K(\mathbf{x}) = c(\mathbf{x}) f(b/c)$. The diffuse attenuation coefficient is an important quantity because it is an *apparent* optical property of the medium and therefore depends on the structure of the incoming light field. It is defined as a ratio so it is easily measurable quantity that does not require absolute measurements. Other apparent and inherent optical properties can be expressed in terms of diffuse extinction. The *optical depth* τ will be defined later in this section.

Phase Function

The phase function $P(\mathbf{x}, \vec{\omega}, \vec{\omega}')$ is the probability that light coming from an incident direction $\vec{\omega}$ will scatter into an exitant direction $\vec{\omega}'$ upon a scattering event at point $\mathbf{x}$. The phase function can be seen as a true probability distribution and is therefore normalized:

$$\int_{4\pi} P(\vec{\omega}, \vec{\omega}')d\omega' = 1.$$

The phase function P only depends on the phase angle $\cos\theta = \vec{\omega} \cdot \vec{\omega}'$ and is reciprocal: $P(\mathbf{x}, \vec{\omega}, \vec{\omega}') = P(\mathbf{x}, \vec{\omega}', \vec{\omega})$.The mean cosine g of the scattering angle is defined as:

$$g = \int_{4\pi} P(\vec{\omega}, \vec{\omega}')(\vec{\omega} \cdot \vec{\omega}')d\omega'.$$

If a mean cosine is 0, the scattering is isotropic. On the other hand, if g is negative, backward scattering dominates; and if g is positive, the scattering is predominantly in the forward direction. The phase function only describes what happens when light is scattered by the particle and does not tell you anything when light gets absorbed upon the scattering event. The shape of the phase function strongly depends on size and orientation of particles in the medium. In general, the phase function will differ from particle to particle. For simplicity and practical reasons, an average phase function that describes the most important features of the scattering process is used. For clarity we will drop positional dependence of optical parameters through the rest of the notes.

ISOTROPIC PHASE FUNCTION. The simplest phase function is the isotropic phase function:

$$P(\vec{\omega}, \vec{\omega}') = \frac{1}{4\pi}. \tag{1}$$

The light will scatter in random direction over the entire sphere with equal probability.

HENYEY-GREENSTEIN PHASE FUNCTION. The Henyey-Greenstein (HG) phase function was first introduced by Henyey and Greenstein [23] to describe scattering of radiation in a galaxy. The Henyey-Greenstein phase function has proven to be useful in approximating the angular scattering dependence of single scattering events in biological tissues, water, clouds and many other natural materials. It is very popular, because it is a fast and simple approximation to true Mie scattering phase function that is very expensive to evaluate. The HG phase function is:

$$P_{HG}(\vec{\omega},\vec{\omega}') = \frac{1-g^2}{(1+g^2-2g\cos\theta)^{\frac{3}{2}}} \tag{2}$$

where the asymmetry parameter g controls the shape of light redistribution upon scattering event. Note that the phase function P_{HG} still needs to be normalized by multiplying it by $1/4\pi$.

DOUBLE HENYEY-GREENSTEIN PHASE FUNCTION. The Henyey-Greenstein phase function cannot capture scattering events that have two lobes, one in forward direction and the other in backward direction. A simple extension has been proposed by Kattawar [34] that combines a forward and backward elongated HG phase functions:

$$P(\vec{\omega},\vec{\omega}') = (1-f)P_{HG}(\vec{\omega},\vec{\omega}',g_1) + fP_{HG}(\vec{\omega},\vec{\omega}',g_2) \tag{3}$$

where $g_1 > 0$ (forward scattering) and $g_2 < 0$ (backward scattering). Further extensions are possible by combining more than two lobes.

SCHLICK PHASE FUNCTION. While the Henyey-Greenstein phase function is a good approximation to Mie scattering, it is still relatively expensive to evaluate. Schlick observed that the accurate shape is often not crucial for rendering applications and he replaced a relatively expensive exponentiation with even simpler expression [7]:

$$P(\vec{\omega},\vec{\omega}') = \frac{1-k^2}{(1+k\cos\theta)^2} \tag{4}$$

where k is a parameter similar to the asymmetry parameter g: $-1 \leq k \leq 1$. The phase function still needs to be normalized by multiplying it by $1/4\pi$, so that it will integrate to 1.

RAYLEIGH PHASE FUNCTION. In order for Rayleigh scattering to be valid, the size of the particle must be much smaller than the wavelength λ of the incident light. Small particles (about 0.1λ) scatter light equally in forward and backward directions:

$$P(\vec{\omega},\vec{\omega}') = \frac{3}{4}\frac{(1+\cos^2\theta)}{\lambda^4}. \tag{5}$$

Unlike phase function that approximate Mie scattering, Rayleigh scattering phase function is inversely proportional to the fourth power of wavelength of light. This

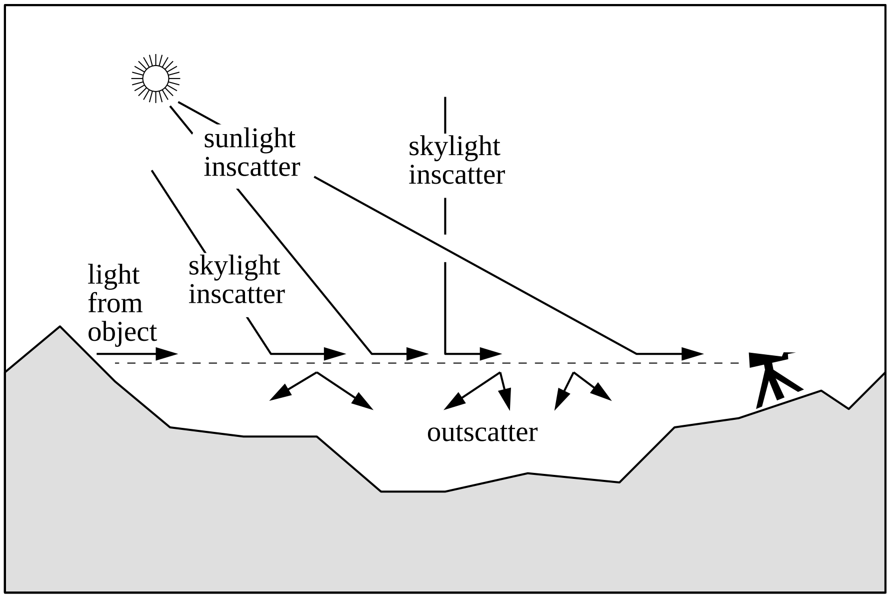

Figure 3: Scattering in participating media. The intensity of light in viewing direction is reduced due to absorption and outscattering. Scattering of light into viewing direction increases the intensity and modifies the color of the light seen by the observer. (Figure courtesy of AJ Preetham)

means that red light (700*nm*) is scattered about ten times less than blue light (400*nm*).

CORNETTE-SHANKS PHASE FUNCTION. Cornette and Shanks [13] modified the Henyey-Greenstein phase function and gave it a more reasonable physical expression:

$$P(\vec{\omega},\vec{\omega}') = \frac{3}{2}\frac{(1-g^2)}{(2+g^2)}\frac{1+\cos^2\theta}{(1+g^2-2g\cos\theta)^{\frac{3}{2}}}. \tag{6}$$

This phase function is especially useful for clouds. Note that if $g = 0$, this function is equivalent to Rayleigh scattering. As before, the phase function needs to be normalized by multiplying it by $1/4\pi$.

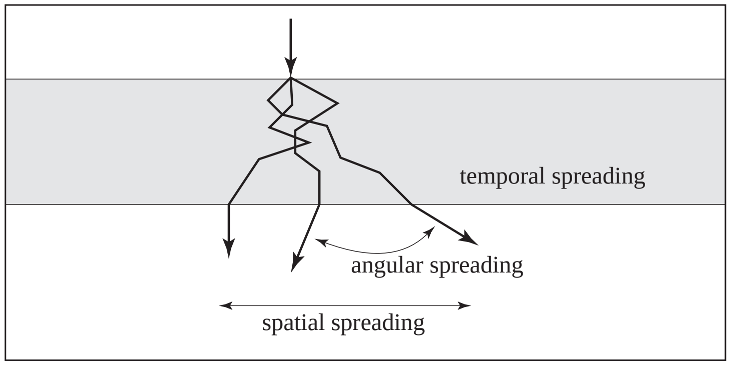

Figure 4: Scattering in a highly scattering medium. Original radiance undergoes a series of scattering events that result in angular, spatial and temporal spreading of the original radiance distribution.

4.1 Light Transport

Upon entering the medium, incoming light undergoes a series of scattering and absorption events that modify both the directional structure of the incoming light field and its intensity. Light intensity in viewing direction $\vec{\omega}$ is reduced (attenuated) due to *absorption* and *outscattering*. On the other hand, as a result of scattering, light can also scatter into a viewing direction (*inscattering* from arbitrary direction and change the light intensity and color in viewing direction. Figure 3 illustrates scattering events and their contributions to the final color and intensity of the light. As a result of multiple scattering events, the original radiance distribution undergoes angular, spatial and temporal spreading which result in different radiance distribution. Figure 4 shows spreading effects in an arbitrary highly scattering medium. Table 1 summarizes terms and quantities used in these notes. We now examine the amount of attenuation and inscattering due to absorption and scattering in arbitrary participating media. We first write the change in radiance when light is moving through a segment of size ds.

ABSORPTION. Absorption coefficient a describes the probability of a photon being absorbed per unit length. The change in radiance dL in direction $\vec{\omega}$ due to absorption is:

$$dL(\mathbf{x}, \vec{\omega}) = -a(\mathbf{x})L((\mathbf{x}, \vec{\omega})ds. \tag{7}$$

OUTSCATTERING. Scattering coefficient b describes the probability of a photon

$\mathbf{x}$	Generic location in $\mathbb{R}^3$
$\vec{\omega}$	Generic direction
$a(\mathbf{x})$	Absorption coefficient at a point
$b(\mathbf{x})$	Scattering coefficient at a point
$c(\mathbf{x})$	Extinction coefficient at a point
g	Mean cosine of the scattering angle
Q	Volume source distribution
$P(\vec{\omega}, \vec{\omega}')$	Phase function
T	Transmittance
τ	Optical depth

Table 1: Notation and quantities used in these notes.

being scattered per unit length. The change in radiance dL in direction $\vec{\omega}$ due to scattering is:

$$dL(\mathbf{x}, \vec{\omega}) = -b(\mathbf{x})L((\mathbf{x}, \vec{\omega})ds. \tag{8}$$

EXTINCTION. The total change in radiance due to absorption and outscattering in direction $\vec{\omega}$ along the segment length ds is:

$$dL(\mathbf{x}, \vec{\omega}) = -c(\mathbf{x})L((\mathbf{x}, \vec{\omega})ds \tag{9}$$

where $c = a + b$ is the attenuation (extinction) coefficient that describes the probability that the photon will be either scattered or absorbed.

INSCATTERING. As mentioned before, the light can scatter into the viewing direction $\vec{\omega}$ from all directions. The change in radiance over segment ds in direction $\vec{\omega}$ is:

$$dL(\mathbf{x}, \vec{\omega}) = b(\mathbf{x}) \int_{4\pi} P(\mathbf{x}, \vec{\omega}, \vec{\omega}')L(\mathbf{x}, \vec{\omega'})d\omega' ds \tag{10}$$

where $P(\mathbf{x}, \vec{\omega}, \vec{\omega}')$ is the phase function. Since it is possible that light can scatter from any direction, the incident radiance must be integrated over entire sphere of directions. In practice, this results in computationally very expensive computation.

EMISSION. It is possible the volumetric medium is also emitting light. The change in radiance dL due to emission withing the medium is:

$$dL(\mathbf{x}, \vec{\omega}) = -a(\mathbf{x})L_e(\mathbf{x}, \vec{\omega})ds \tag{11}$$

where $L_e(\mathbf{x},\vec{\omega})$ is the emitted radiance at point $\mathbf{x}$ in direction $\vec{\omega}$.

Optical Depth and Transmittance. Optical depth τ over a uniform segment of length ds is a product of extinction coefficient c and segment length ds. Optical depth $\tau(s)$ over a segment of length s in arbitrary inhomogeneous is then:

$$\tau(s) = \int_0^s c(\mathbf{x}+s'\vec{\omega})ds'. \tag{12}$$

More generally, the optical depth $\tau(\mathbf{x},\mathbf{x}')$ defined over an arbitrary line segment starting at parameter s and ending at parameter s' is:

$$\tau(s,s') = \int_s^{s'} c(\mathbf{x}+t\vec{\omega})dt. \tag{13}$$

Optical depth related to transmittance $T(s,s')$ over a line segment from s to s' as follows:

$$T(s,s') = \exp(-\tau(s,s')). \tag{14}$$

The transmittance can be interpreted as the percentage of light that reached point $\mathbf{x}'$ at parameter s' starting at point $\mathbf{x}$ and parameter s. Opacity is just an inverse of the transmittance $T(s,s')$:

$$\alpha(s,s') = 1 - T(s,s'). \tag{15}$$

Light Transport Equation. We have so far described the change in radiance dL over distance ds due to absorption, outscattering, emission and inscattering. By combining all these components, the total change in radiance $L(\mathbf{x},\vec{\omega})$ at point $\mathbf{x}$ and in direction $\vec{\omega}$ is written in terms of the *light transport equation* [2, 26]:

$$\begin{aligned}(\vec{\omega}\cdot\nabla)L(\mathbf{x}+s\vec{\omega}) &= -c(\mathbf{x})L(\mathbf{x},\vec{\omega}) + b(\mathbf{x})\int_{4\pi} P(\mathbf{x},\vec{\omega},\vec{\omega}')L(\mathbf{x},\vec{\omega}')d\omega' \\ &\quad + a(\mathbf{x})L_e(\mathbf{x},\vec{\omega})\end{aligned} \tag{16}$$

It is often convenient to split the total radiance within the medium into components and write it as the sum of *unscattered* (direct) radiance L_{un}, the *emission* L_e and the *scattered* radiance L_{sc}:

$$L(\mathbf{x},\vec{\omega}) = L_{un}(\mathbf{x},\vec{\omega}) + L_{sc}(\mathbf{x},\vec{\omega}) + L_e(\mathbf{x},\vec{\omega}). \tag{17}$$

Here L_{un} is the radiance which intensity has been reduced due to absorption and outscattering along the length S. L_{sc} is the radiance that has undergone a series of scattering events and finally scattered into a small cone around the observation direction $\vec{\omega}$.

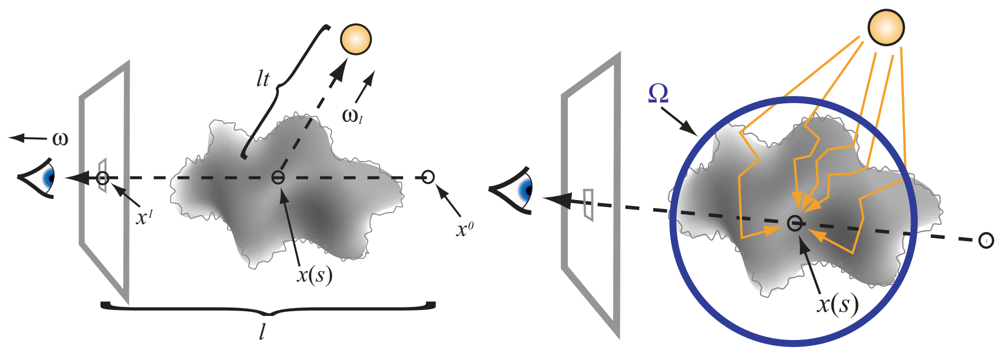

Figure 5: Monte Carlo Ray Tracing solution of the light transport equation in an arbitrary inhomogeneous medium. Left: direct lighting computation. For every sample point ong the viewing ray, a ray toward each light source is sent and light intensity is computed via raymarching. Right: Inscattering computation. At every sample point on the viewing ray, indirect light contribution is integrated over entire sphere of directions. (Figures from Kniss *et. al.* [36])

4.2 Solving Light Transport Equation Monte Carlo Ray Tracing

Analytic solutions for light transport equation for inhomogeneous media expressed in equation 17 are not possible. Numerical solutions are necessary to solve this integral equation. There are many different methods of solving this equation ranging from very robust Monte Carlo methods to very specialized solutions that make additional assumptions about optical properties of the media and boundary conditions.

Monte Carlo ray tracing is an accurate algorithm for solving the radiative transfer equation in arbitrary media. We march through the medium in direction $\vec{\omega}$ sampling points along the ray. At every sampling point along the ray, we send a a ray toward each light source. The contribution of each light source is computed by marching along a ray toward light direction $\vec{\omega}$ and computing attenuation (see Figure 5, left). Then the light from previous step is attenuated and the light that is inscattered into the viewing direction $\vec{\omega}$ is gathered (see Figure 5, right).

DIRECT LIGHTING. The light that has been scattered in the participating media *exactly* once on the way from a light source to the viewer is the *direct lighting*. Using standard ray marching, the direct lighting portion of equation 17 is:

$$L_{n+1}(\mathbf{x}+\Delta s\vec{\omega},\vec{\omega}) = \sum_{l}^{allNlights} L_l(\mathbf{x},\vec{\omega}_l')P(\mathbf{x},\vec{\omega},\vec{\omega}_l')b(\mathbf{x})\Delta s + \\ + e^{-c(\mathbf{x})\Delta s}L_n(\mathbf{x},\vec{\omega}) \quad (18)$$

where $L_l(\mathbf{x},\vec{\omega}_l')$ is the contribution from light source l, and Δs is the step say in ray march. The light intensity $L_l(\mathbf{x},\vec{\omega}_l')$ is computed by shooting a *shadow ray* towards each light source, ray marching through the volume until the light source is hit and computing transmittance T (equation 14) along the shadow ray:

$$L_l(\mathbf{x},\vec{\omega}_l') = TI_l(\vec{\omega}_l') \quad (19)$$

where $I_l(\vec{\omega}_l')$ is the intensity of the light source in direction $\vec{\omega}_l'$.

INDIRECT LIGHTING. The light that has scattered multiple times (inscattered light) is collected recursively for each ray:

$$L_{n+1}(\mathbf{x}+\Delta s\vec{\omega},\vec{\omega}) = \left(\frac{4\pi}{M}\sum_{i=1}^{M} L_{sc}(\mathbf{x},\vec{\omega}_i)P(\mathbf{x},\vec{\omega},\vec{\omega}_i)\right)b(\mathbf{x})\Delta s \quad (20)$$

where M is the number of directional samples taken and Δs is the ray marching step size. Computing indirect contribution involves integrating over M directions. The light contribution from each direction involves recursive computation that grows exponentially.

COMPLETE LIGHTING COMPUTATION. By combining the direct and indirect contributions, we compute the total radiance L in participating media:

$$L_{n+1}(\mathbf{x}+\Delta s\vec{\omega},\vec{\omega}) = \sum_{l}^{allNlights} L_l(\mathbf{x},\vec{\omega}_l')P(\mathbf{x},\vec{\omega},\vec{\omega}_l')b(\mathbf{x})\Delta s + \\ \left(\frac{4\pi}{M}\sum_{i=1}^{M} L_{sc}(\mathbf{x},\vec{\omega}_i)P(\mathbf{x},\vec{\omega},\vec{\omega}_i)\right)b(\mathbf{x})\Delta s + \\ e^{(-c(\mathbf{x})\Delta s)}L_n(\mathbf{x},\vec{\omega}).$$

Equation 21 computes direct contribution, indirect contribution and adds contribution from the previous segment L_n.

While Monte Carlo ray tracing is robust and powerful, it is also very slow because of the large number of rays needed. At evry sampling point along the ray, exponential number of rays is spawned in order to compute inscattering. There are many improvements over the basic brute fore Monte Carlo ray tracing algorihtm just described that improve both convergence rate (adaptive raymarching based on material's density) and quality (Russian roulette, importance sampling, etc.).

Monte Carlo methods are also often used to compute radiative transport within a medium. Although simple and powerful, these methods suffer from slow convergence. Finite element methods are also used, but they require large amounts of storage to capture discontinuities and strong directional light distributions. A brief overview of many existing methods is presented in Section 5. Solutions specially tailored for computer graphics applications for efficient implementation on modern graphics hardware will be presented in later chapters of these course notes.

5 Background

A vast amount of literature exists on scattering and light transport. A non-linear integral scattering equation that describes the scattering events inside a volume has been studied extensively by Ambarzumian [1], Chandrasekhar [12], Bellman *et. al.* [4] and van de Hulst [68]. Their work ranges in complexity from semi-infinite homogeneous isotropic atmospheres to finite inhomogeneous anisotropic atmospheres. Mobley [44] applied these one-dimensional scattering equations to a variety of problems, mainly in hydrologic optics. Pharr and Hanrahan [53] described a mathematical framework for solving this scattering equation in the context of computer graphics and a variety of rendering problems and also described a numerical solution in terms of a Monte Carlo sampling method. Pharr and Hanrahan exploited interaction principle which encapsulates all transfer properties within a layer. Adding and doubling method extends interaction between more than two homogeneous layers [20].

Siegel and Howell [60] provide a fundamental description of light transport as a classic equation of transfer. In a seminal work, Blinn [9] presented a model for the reflection and transmission of light through thin clouds of particles based on probabilistic arguments and single scattering approximations in which Fresnel effects were considered. He recognized the importance of light transport for computer graphics applications. The first methods for solving light transport in participating media for computer graphics only accounted for direct illumination

(Max [43], Klassen [35]). Analytical approximations to the light transport equation exist, but they are severely restricted by underlying assumptions such as homogeneous optical properties and density, simple lighting, or unrealistic boundary conditions. Numerical methods and algorithmic approaches are needed to address the global illumination in environments including participating media and volumetric materials. We briefly review several different methods. Perez *et. al.* [52] survey and classify global illumination algorithms in participating media in more detail. An alternative description of light propagation was done by Pharr and Hanrahan [53] who described a mathematical framework for solving the scattering equation in the context of a variety of rendering problems and also described a numerical solution in terms of a Monte Carlo sampling method.

Monte Carlo Methods

Monte Carlo methods are robust and widely used techniques for solving light transport equation. Hanrahan and Krueger modeled scattering in layered surfaces with linear transport theory and derived explicit formulas for backscattering and transmission [21]. Their model is powerful and robust, but it relies on Monte Carlo methods and therefore suffers from noise artifacts and slow convergence. Blasi *et. al.* [8, 7] described an algorithm based on a particle light tracing simulation. The interaction points in the media are spaced at a constant distance. Similarly, Pattanaik and Mudur [50] also presented a Monte Carlo light tracing algorithm. Their method generates random walks starting from the light source, and interaction points in the medium are sampled according to transmittance of the volume. Lafortune and Willems [38] improved upon the method by tracing paths both from light sources and the eye. Baranoski and Rokne [3] simulated light transport in leaves using the Monte Carlo method. Jensen and Christensen [30] presented a two pass volume photon density estimation method using a photon map. Their method is simple, robust and efficient but suffers from additional memory requirements to store photons if the extent of the scene is large or the lighting configuration is very difficult. Dorsey *et. al.* [18] described a method for full volumetric light transport inside stone structures using a volumetric photon map representation. Photon map was also used for depicting scattering in wet materials [31], smoke [19] and fire [45]. Veach and Guibas [69] proposed a novel global illumination algorithm that found an important path that contributed the most to the final pixel intensity by Markov Chain Monte Carlo method. Once the important path was found, the path space was explored locally because it was likely that other important paths would be nearby. Pauly *et. al.* [51] extended the method for participating media and proposed suitable mutation strategies for paths. Although extremely general and robust, as it could handle any lighting condition and configuration, it still suffered from classical Monte Carlo artifacts.

Finite Element Methods

Finite element methods provide an alternative approach to solving integral equations. Rushmeier [59, 58] presented zonal finite element methods for isotropic scattering. Bhate [6] described an improvement over the zonal method that included a progressive refinement of elements. Sillion [61] extended the classical hierarchical radiosity algorithm to include isotropically scattering participating media. Spherical harmonics were also used by Kajiya and von Herzen [33] to compute anisotropic scattering in volumetric media while Languenou *et. al.* [39] used discrete ordinate methods. Bhate [5] extended the zonal method to include the interactions between surface and volume elements that were not accounted for by Kajiya and von Herzen [33]. Patmore [49] formulated a local solution for non-emitting volumes and the global solution was found by iterative expansion of local solutions on a cubic lattice. Max *et. al.* [42] used a one-dimensional scattering equation to compute the light transport in tree canopies by solving a system of differential equations by application of the Fourier transform. Their method became expensive for forward peaked phase functions as the hemisphere needed to be more finely discretized. All of these finite element methods require discretization of volumetric media in space and angles, and therefore require a large amount of memory to effectively compute interactions between elements, especially if discontinuities or glossy reflections are to be captured.

Other methods

Alternative methods that do not rely on Monte Carlo or finite element methods have also been proposed. Stam [64] presented a solution to multiple scattering by solving the diffusion equation using a multigrid method. Jensen *et. al.* [32] introduced an analytical diffusion approximation to multiple scattering, which is especially applicable for materials that exhibit considerable subsurface light transport. Their method relies on the assumption that the multiply-scattered light is nearly isotropic and cannot be easily extended to inhomogeneous materials. Lensch *et. al.* [40] implemented this method in graphics hardware and Jensen and Buhler [29] extended this diffusion approximation to be computationally more efficient by storing illumination in a hierarchical grid.

There have also been some specialized approximations that are not applicable to arbitrary participating or volumetric media. Nishita *et. al.* [48] presented an approximation to light transport inside clouds. Similarly, Irwin uses adaptive Simpson quadrature method to compute sky radiance while only accounting for Rayleigh scattering [25]. Jackel and Walter presented a method of renndering sky based on Mie scattering using extinction correction method to deal with multiple scattering [28]. Harris *et. al.* [22] described a fast hardware accelerated method for realistic depiction of clouds. Several other hardware algorithms for approximating light transport in volumetric media has been described by Nishita *et. al.* [47], Dobashi *et. al.* [16] and Iwasaki *et. al.* [27]. Premože and Ashikhmin [56] presented a model for light transport in water. Their approximation was specialized in that it could only be applied to natural water bodies. Nishita [46] presented an overview of light transport and scattering methods for natural environments. Stam described an efficient but highly specialized illumination model for a skin layer [65]. Preetham *et. al.* employed Monte Carlo simulations for sky simulations [55]. The results of simulations were then fit to a parametric model to obtain a practical model of a daylight sky. Dobashi et al. [17] proposed a fast method for rendering the atmospheric scattering effects based look-up tables that store the intensities of the scattered light, and these are then used as textures. Sloan *et. al.* precomputed radiance transfer in low frequency illumination environment and stored the transferred radiance using spherical harmonics basis functions [62]. Kniss *et. al.* proposed an empirical volume shading model accounting for scattering in translucent materials by blurring illumination within a cone [36].

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