Modeling Skylight and Aerial Perspective

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Theory of Scattering in Atmosphere

Atmosphere

Earth is enveloped by a vast amount of air called the atmosphere. It is relative thin compared to the size of the earth and fades away with increasing distance from the earth's surface. The earth's atmosphere is categorized into 4 layers - troposphere, stratosphere, mesosphere and thermosphere. Troposphere where all the weather takes place and stratosphere, the two closest layers to earth's surface constitute more than 99% of the atmosphere. The troposphere and stratosphere extend up to 12 km and 53 km respectively from the earth's surface. The earth's atmosphere is composed of many gases. Nitrogen from decay of biological products constitutes 98% and Oxygen from photosynthesis constitutes 21% of the atmosphere. Gravity holds the atmosphere close to the earth's surface and explains why the density of the atmosphere decreases with altitude.

The density and pressure of the atmosphere vary with altitude and depends on solar heating and geomagnetic activity. The simplest is an exponential fall off model where pressure and density decrease exponentially with altitude. In 1976, US Standard Atmosphere Model was adopted by COESA (Committee on Extension to the Standard Atmosphere) which describes the earth's atmosphere as composed of 7 layers up to 86 km and pressure, temperature and density are specified for each layer [McCartney1976]. The density is calculated using a perfect gas relationship.

In addition to the various gases, atmosphere also contains water vapor, dust particles, etc. The molecules and particles absorb energy at discrete wavelengths, which are determined by their internal properties. For example, molecular oxygen and ozone absorbs light in the ultra violet spectrum. Water vapor, methane, nitrous oxide, ozone, and CO_2 absorb light in the infrared range.

In addition to absorption, molecules and particles also scatter energy out from its original direction. Sun’s white light is scattered once (primary scattering) or multiple times (secondary scattering) into the viewing ray as shown in Figure 1. The scattered light is received at the earth’s surface from all directions as diffuse skylight or daylight.

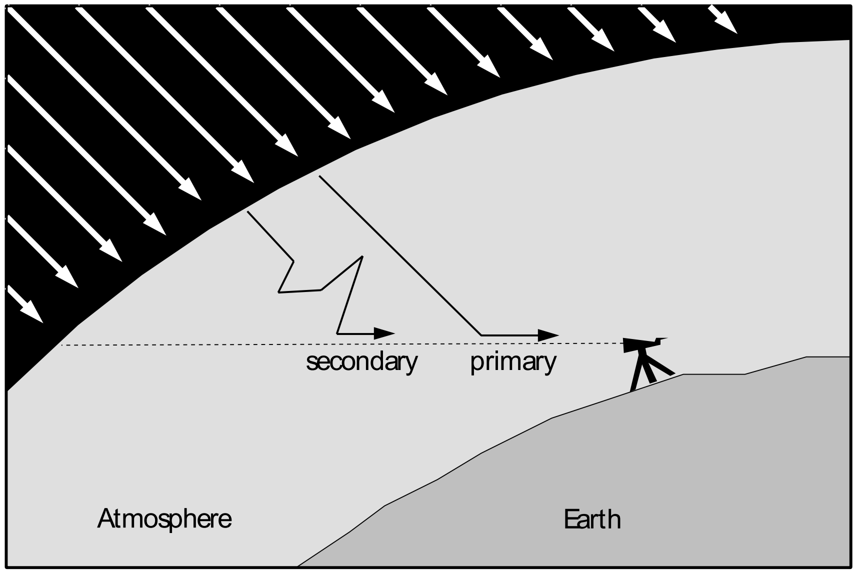

Figure 1: Scattering of sunlight in atmosphere.

Earth's surface is not flat and this plays a very important role in atmospheric optics. *Optical mass* of a path is defined as the mass of the medium in that path of unit cross-section and is given by

$$m = \int_0^s \rho(x)dx,$$

where $\rho(x)$ is the density of medium. *Optical length* for a path is defined as the optical mass divided by the density at the earth's surface ρ_0 and is given by

$$l = \frac{1}{\rho_0}\int_0^s \rho(x)dx$$

Optical length has dimensions of length. The optical length in the zenith direction for molecules is 8.4 km and for aerosols is 1.25 km. Figure 2 shows the optical lengths for different directions in the atmosphere.

The relative optical length is defined as the ratio of the optical length of any path to optical length at zenith direction and is given by following approximation [Iqbal1983]

$$\frac{l(\theta_s)}{l_{zenith}} = \frac{1}{\cos\theta_s + 0.15(93.885 - \theta_s)^{-1.253}},$$

where θ_s is the angle from zenith in degrees.

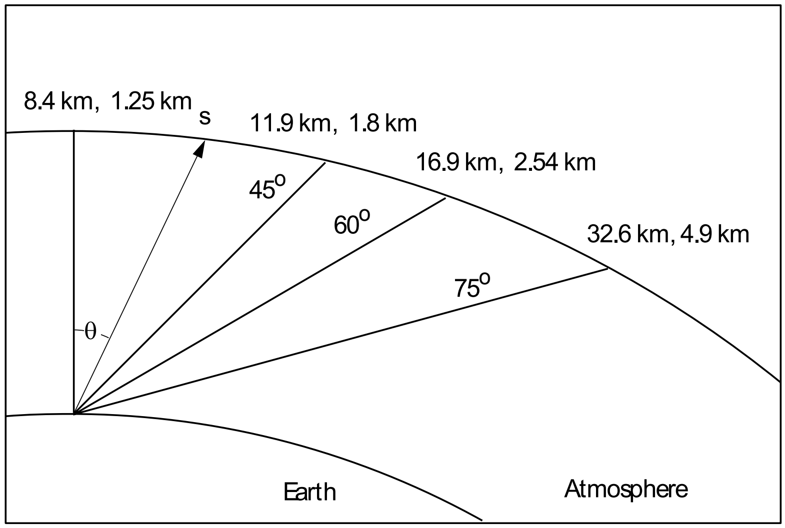

Figure 2: Optical lengths for molecules and aerosols for different paths in the atmosphere.

The sun's light travels through a much larger atmosphere when it is close to the horizon. Therefore, a larger amount of blue light is scattered away causing the sun to appear orange-red. Increased atmosphere presence in the horizon direction makes the stars appear brighter in the zenith direction compared to the horizon direction.

Scattering

Scattering is a process by which a particle redistributes a fraction of the incident energy into a total solid angle. The scattering properties depend on the refractive index and size of the particles.

It is common for smaller particles to scatter uniformly in the forward and backward directions and for larger particles to scatter strongly in the forward directions. Scattering by one particle is independent of the other as long as the distance between the particles are greater than the particle size. This is known as *independent scattering*. A scattering event for the first time is known as first order scattering. Scattered light may be scattered again by another particle and is said to undergo second order scattering. In reality, sun's light is scattered multiple times in the atmosphere.

The amount of scattering is linearly proportional to the density of the atmosphere, which varies with altitude. A choice of an analytic exponential model or a lookup table based on US Standard Atmospheres for density is available for use.

Rayleigh scattering

Particles smaller than the wavelength and usually less than 0.1 times the wavelength of light exhibit Rayleigh scattering [Rayleigh1871]. Discovered by the Nobel Prize winner Lord Rayleigh, Rayleigh scattering is observed by molecules in the earth's atmosphere. The amount of scattering for such particles is inversely proportional to the 4^{th} power of the wavelength. These particles scatter equally in the forward and backward directions. The total scattering coefficient β and the angular scattering coefficient $\beta(\theta)$ are given by

$$\beta = \frac{8\pi^3 (n^2 - 1)^2}{3N\lambda^4} (\frac{6 + 3p_n}{6 - 7p_n}),$$

$$\beta(\theta) = \frac{\pi^2 (n^2 - 1)^2}{2N\lambda^4} (\frac{6 + 3p_n}{6 - 7p_n})(1 + \cos^2\theta),$$

where n is the refractive index of air and is 1.0003 in the visible spectrum, N is the number of molecules per unit volume and is 2.545×10^{25} for air at standard temperature and pressure, p_n is the depolarization factor with a value of 0.0035 standard for air. The total scattering coefficient for blue light (400 nm) is $2.44 \times 10^{-5} m^{-1}$, for green light (530 nm) is $1.18 \times 10^{-5} m^{-1}$ and for red light (700 nm) is $6.95 \times 10^{-6} m^{-1}$. This means that the blue light is scattered more than the red light, which explains the blue color of the sky and the red color of the sun at low altitudes.

Angular scattering coefficient is also equivalent to the total scattering coefficient times the phase function. The phase function for Rayleigh scattering $f_{air}(\theta)$ is given by

$$f_{air}(\theta) = \frac{3}{16\pi}(1 + \cos^2\theta)$$

Mie Scattering

Larger particles scatter strongly in the forward direction and this scattering phenomenon is called Mie scattering named after Gustav Mie. The scattering is inversely proportional to the second order of the size of the particles and is independent of wavelength. The phase function for angular scattering was approximated by Henyey-Greenstein and is given by the following equation [Henyey1941].

$$f_{HG}(\theta) = \frac{1}{4\pi} \frac{1 - g^2}{\left(1 - 2g\cos\theta + g^2\right)^{3/2}}.$$

Positive values of g represent forward scattering and negative values of g represent backward scattering. The total scattering coefficient is given by

$$\beta = 0.434 c\pi (\frac{2\pi}{\lambda})^{v-2} K,$$

where c is the concentration factor and varies around 6×10^{-17} to 25×10^{-17} as the turbidity increases from clear to overcast, v is the Junge's exponent and a value of 4 is standard for sky model, and K varies from 0.656 for 400 nm to 0.69 for 770 nm. For more details, see [Bullrich1964].

Skylight Models

The atmosphere scatters sun's light multiple times and scattered light is received at the earth's surface in all directions collectively known as skylight.

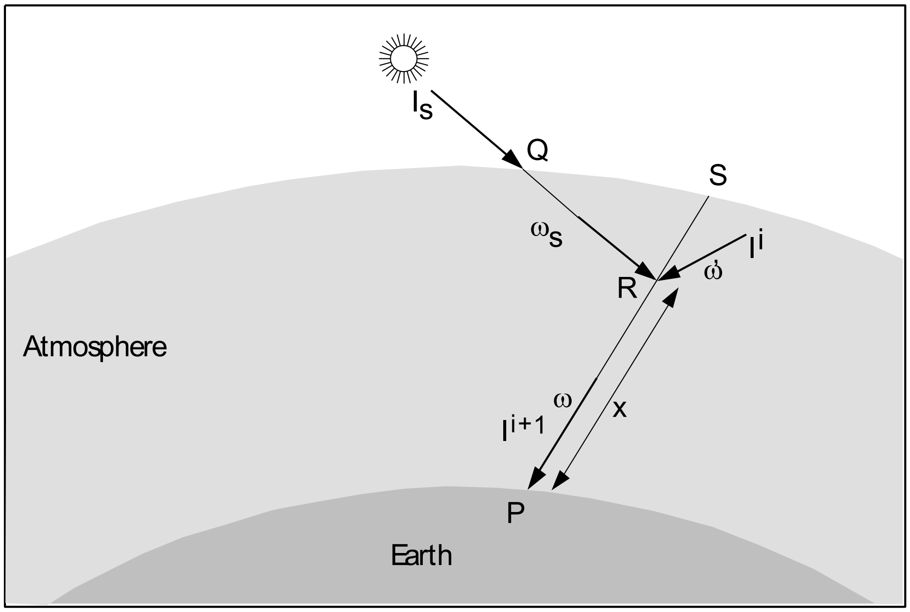

Figure 3: The variables used in skylight computation.

In Figure 3, we would like to compute the skylight in the viewing direction (ω) PS. R is a variable point on PS and RQ is the direction (ω_s) of sunlight. Points Q and S are at the top of the atmosphere. Light from the sun is attenuated as it travels the distance QR, is scattered by particles at R in the direction RP, and then attenuated as it travels a distance RP. Let l_{AB} denote the optical length from A to B. The differential amount of light reaching point P through the path QRP, scattered by a differential volume R is given by

$$dI = E_s e^{-\beta l_{QR}} \beta(\omega, \omega_s) e^{-\beta l_{RP}} dx\,,$$

where E_s is the irradiance of the sun outside the earth's atmosphere, β is the total scattering coefficient, $\beta(\omega,\omega_s)$ is the angular scattering coefficient between directions ω and ω_s and dx is the differential optical length. The total light received at P from all points R on PS is given by

$$I^1(\omega) = \int_P^S E_s e^{-\beta l_{QR}} \beta(\omega, \omega_s) e^{-\beta l_{RP}} dx\,.$$

Superscript "1" on $I(\omega)$ is used to indicate first order scattered light. Light from the sun is scattered more than once before reaching the earth's surface. Let $I^i(\omega')$ be the light reaching point R from direction ω' after being scattered i times. Light scattered into viewing direction ω from all directions ω' at point R is denoted by $S(\omega,x)$ and is given by the integral

$$S(\omega, x) = \int_0^{4\pi} I^i(\omega') \beta(\omega, \omega') d\omega'$$

The light scattered in direction ω at R is attenuated as it travels the distance RP through the atmosphere. Light received at earth's atmosphere after i+1 scattering events is given by

$$I^{i+1}(\omega) = \int_P^S e^{-\beta l_{RP}} S(\omega, x) dx$$

$$I^{i+1}(\omega) = \int_P^S e^{-\beta l_{RP}} \int_0^{4\pi} I^i(\omega')\beta(\omega,\omega')d\omega' dx$$

Starting from the equations for $I^1(\omega)$ and $I^{i+1}(\omega)$ in terms of $I^i(\omega)$, one can compute $I^2(\omega)$, $I^3(\omega)$ and so on, where $I^2(\omega)$ and $I^3(\omega)$ is the second and third order scattered light into viewing direction ω. Total light scattered into the viewing ray $I(\omega)$ is the sum of first order, second order and higher orders of scattered light and is given by

$$I(\omega) = I^1(\omega) + I^2(\omega) + I^3(\omega) + \cdots$$

The above equations assume only one kind of particles in the atmosphere. Real atmosphere consists a number of different kinds of particles, of most importance being molecules and aerosols. The above equations can be extended to multiple particles and is left as an exercise for the reader.

Simulation Based Methods

All simulation methods are based on the general atmosphere equations presented above. The variations arise from use of different models for atmosphere density, different atmosphere composition, scattering coefficients etc.

One of the first models for atmospheric scattering was presented by Klassen [Klassen1987] which is a must read for anyone interested in modeling atmospheric scattering. He used a simple constant density atmosphere model on a flat earth surface. Flat earth model performs poorly for skylight computations on viewing rays close to the horizon. For aerial perspective discussed later, where distances viewed are usually of the order of a few tens of kilometers, the flat earth model is a good approximation.

Kaneda et al employed similar concepts to Klassen and used a more realistic atmospheric model for his simulations [Kaneda1991]. He modeled a spherical earth with an exponential decay density distribution.

Nishita et al [Nishita1996] take a step closer to reality and model higher orders of scattering, which is responsible for the whitening effect of the sky. He proposes a fast method for single scattering computations.

Analytic Models and Approximations

Simulation based methods are computationally very expensive making it unusable for all practical purposes. Analytic or approximate models for skies that represent simulated data or real world atmospheric data collected over the years is preferable for ease of use in computer graphics. Of all real skies, the two extremes are clear and overcast skies.

CIE Overcast sky luminance model

An overcast sky is equivalent to a dark sky with plenty of clouds. Moon and Spencer proposed a luminance model for overcast skies. This was later simplified and adopted by the International Commission on Illumination - abbreviated as CIE from its French title Commission Internationale de l'Eclairage [CIE1994]. The overcast sky luminance Y_{OC} is given by

$$Y_{OC} = Y_z \frac{1 + 2\cos\theta}{3},$$

where θ is the angle from the zenith and Y_z is the zenith for overcast skies and can be obtained from analytic formulas adopted by CIE [CIE1994].

CIE Clear sky luminance model

Pokrowski proposed a sky luminance model from theory and sky measurements for clear sky. Kittler improved this clear sky model, which was later adopted by the CIE for the clear sky model in 1973 [CIE1994]. The clear sky luminance Y_C is given by

$$Y_C = Y_z \frac{(0.91 + 10e^{-3\gamma} + 0.45\cos^2\gamma)(1 - e^{-0.32/\cos\theta})}{(0.91 + 10e^{-3\theta_s} + 0.45\cos^2\theta_s)(1 - e^{-0.32})},$$

where Y_z is the zenith luminance and the angles are defined in Figure 4.
The CIE luminance models provide us with relative luminance and not absolute luminance.

ASRC-CIE model luminance model

ASRC-CIE model is a linear combination of four luminance models - the standard CIE cloudless sky, a high turbidity formulation of the latter, a realistic formulation for intermediate skies proposed by Nakamura and the standard CIE overcast sky. The sky clearness and sky brightness factors are used to determine the weights for these four skies [Littlefair1994].

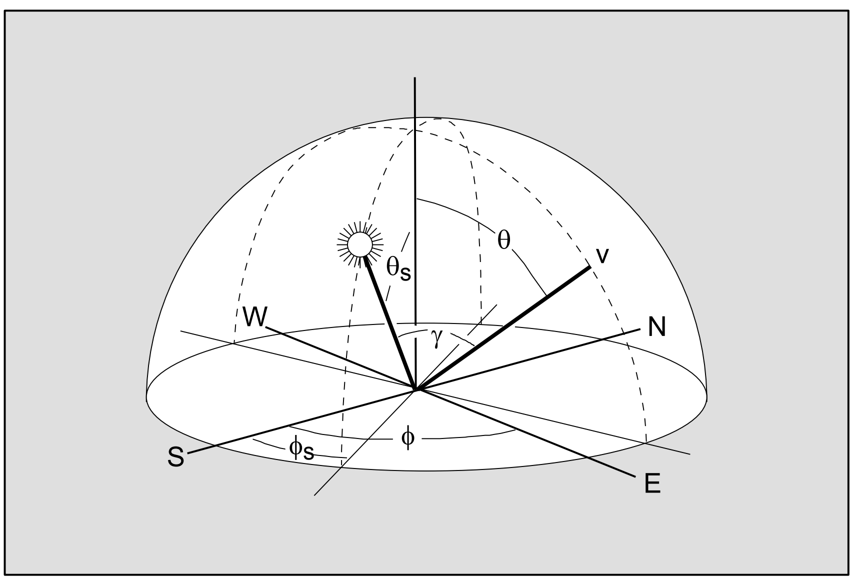

Figure 4: Angles and directions on the sky dome [Preetham1999].

Perez all weather luminance model

Perez et al proposed an all weather sky luminance model [Perez1993]. This model was based on 5 different parameters, which related to darkening or brightening of the horizon, luminance gradient near the horizon, relative intensity of the circumsolar region, width of the circumsolar region and relative backscattered light. The Perez model is given by

$$F(\theta,\gamma) = (1 + Ae^{B/\cos\theta})(1 + Ce^{D\gamma} + E\cos^2\gamma),$$

where A, B, C, D and E are the distribution coefficients and γ and θ are the angles shown in Figure 4. The Perez model sky luminance Y_P is given by

$$Y_P = Y_z \frac{F(\theta,\gamma)}{F(0,\theta_s)}.$$

Perez model is similar to CIE model and has been found to be more accurate and has also been validated by Ineichen [Ineichen1994].

Preetham et al spectral radiance model

Similar to the Perez's all weather luminance model, Preetham et al proposed an analytical model for spectral radiance of sky [Preetham1999]. This model shows variation from clear to overcast sky through a parameter called *turbidity*. Analytic model was arrived from atmospheric simulations using US Standard Atmospheres, and modeling skylight up to second order scattering. Luminance Y and chromaticity values x and y are given by

$$Y = Y_z \frac{F(\theta,\gamma)}{F(0,\theta_s)},$$

$$x = x_z \frac{F(\theta,\gamma)}{F(0,\theta_s)}, \text{ and}$$

$$y = y_z \frac{F(\theta,\gamma)}{F(0,\theta_s)},$$

where $F(\theta, \gamma)$ is from Perez's model with different values of A, B, C, D and E for Y, x and y. The distribution coefficients for luminance and chromaticity values x and y, absolute values for zenith luminance Y_z, zenith chromaticity x_z and y_z are all given in [Preetham1999]. Conversion of chromaticity values into spectral radiance is given by the following standard method for D-illuminants [Wyszecki1982]. The relative spectral radiant power $S_D(\lambda)$ is given by

$$S_D(\lambda) = S_0(\lambda) + M_1 S_1(\lambda) + M_2 S_2(\lambda),$$

where $S_0(\lambda)$ is the mean spectral radiant power and $S_1(\lambda)$ and $S_2(\lambda)$ are the first two eigen vector functions used in daylight illuminants. M_1 and M_2 are functions of chromaticity values x and y and are given by

$$M_1 = \frac{-1.3515 - 1.7703x + 5.9114y}{0.0241 + 0.2562x - 0.7341y},$$

$$M_2 = \frac{0.0300 - 31.4424x + 30.0717y}{0.0241 + 0.2562x - 0.7341y}.$$

A scene rendered using this model is shown in Figure 5.

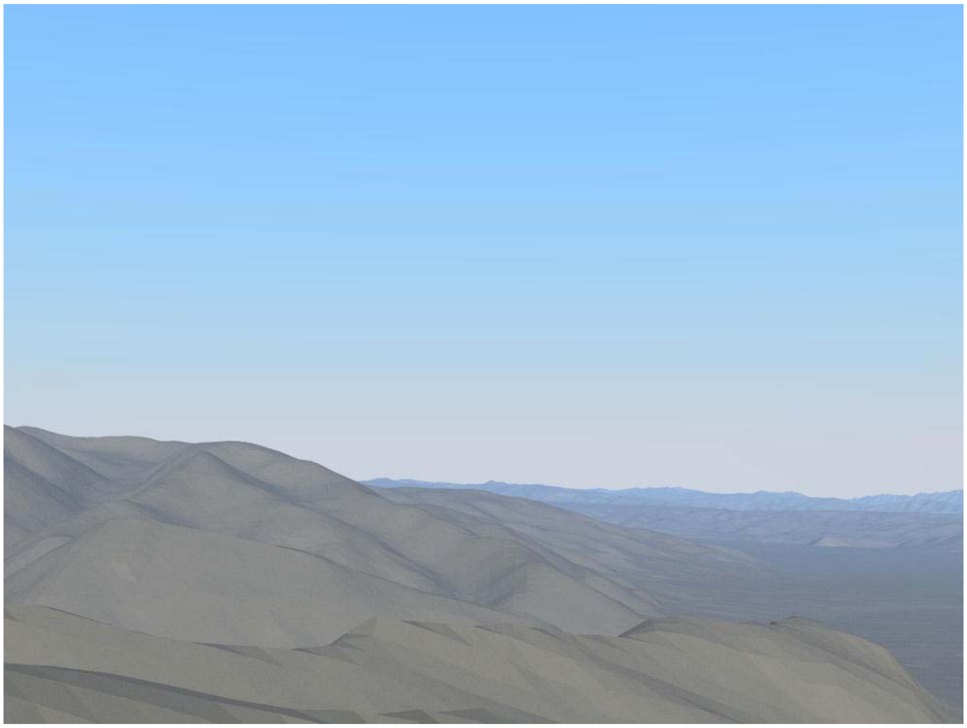

Figure 5: Spectral radiance model for sky [Preetham1999].

Aerial Perspective Model

Many artists have recorded the effect of distant mountains fading away in history. In the presence of a medium such as atmosphere distant objects appear blue. This shift in color helps us in perceiving depth and is known as *aerial perspective*.

Distant objects appear hazier and this is attributed to scattering and absorption along the viewing ray as light travels from the source to the viewer. The light from the source loses intensity and undergoes a spectral shift as scattering and absorption depend on wavelength. In addition to this loss of light, light from other sources like sun, sky and ground are scattered into the viewing ray.

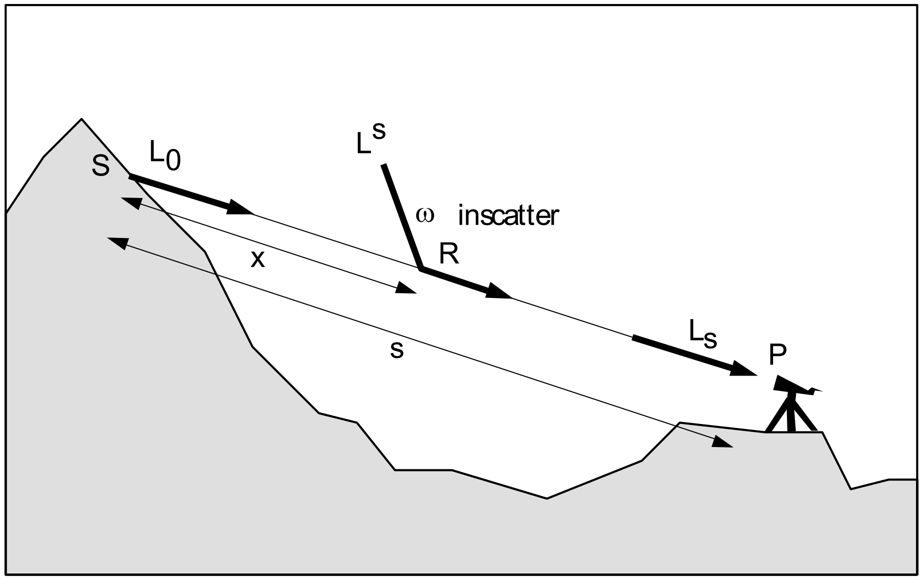

Figure 6: The variables in aerial perspective computation.

In Figure 6, L_0 is the radiance of the distant hill and L_s is the radiance of the ray at the viewer. If f is the extinction factor and L_{in} is the in-scattered light as L_0 travels a distance s to the eye, then

$$L_s = fL_0 + L_{in}$$

The extinction factor f for light traveling the path PS with optical length l_{PS} is given by

$$f = e^{-\beta l_{PS}}$$

Let $L^s(\omega)$ denote the spectral radiance of the sun and sky in the direction ω. Let $S(\omega, x)$ be the term that denotes the light scattered from direction ω into the viewing direction at point R. Therefore,

$$S(\omega, x) = \int_0^{4\pi} L^s(\omega')\beta(\omega, \omega')d\omega'$$

The total light scattered into viewing direction at point R is

$$L_{in} = \int_P^S e^{-\beta l_{RP}} S(\omega, x)dx\,,$$

$$L_{in} = \int_P^S e^{-\beta l_{RP}} \int_0^{4\pi} L^s(\omega')\beta(\omega, \omega')d\omega' dx\,.$$

For landscape scenes that focus on aerial perspective, the viewing rays are close to the earth's surface and it can safely be assumed that the density of the medium is a constant and is equal to that at the earth's surface. For such rays, the optical length l_{AB} is equal to the distance AB. Therefore, l_{PS} is equal to s, l_{RP} is equal to (s-x) and the above equations simplify to

$$f = e^{-\beta s} \text{ and}$$

$$L_{in} = \int_0^s e^{-\beta(s-x)} \int_0^{4\pi} L^s(\omega')\beta(\omega, \omega')d\omega' dx\,.$$

The primary contribution to in-scattered light is the direct light from the sun. Therefore, we can safely ignore second order scattering i.e. the light from the sky without loss of quality. Therefore, if E^s is the irradiance from the sun at the earth's surface and ω^s is the sun's direction, then

$$\int_0^{4\pi} L^s(\omega')\beta(\omega,\omega')d\omega' = E^s\beta(\omega,\omega^s)\ .$$

And the total in-scattered light simplifies to

$$L_{in} = \int_0^s e^{-\beta(s-x)} E^s \beta(\omega,\omega^s)dx\ ,$$

$$L_{in} = E^s \frac{\beta(\omega,\omega^s)}{\beta}(1-e^{-\beta s})\ .$$

While the above equations for aerial perspective are valid for scattering by one kind of particles, they can easily be extended to many kinds of particles, and is left as an exercise for the reader.

Accelerated Techniques for Modeling Sky and Aerial Perspective

Accurate calculation of the sky color is expensive because of numerical integration of scattered light along the viewing ray. Modeling higher orders of scattering results in double and triple integration and the computational cost increases exponentially. Dobashi et al discuss various ways to represent and evaluate the sky intensity across the hemispherical domain for various altitudes of the sun [Dobashi1995]. A simple and naive approach would be to represent the sky dome by a mesh grid of sample points. Skylight intensity at any point on the hemisphere can be obtained by linearly interpolating between these sampled points. Any function can be represented by the summation of a set of basis functions and another approach to representing the sky function over the hemisphere is to use spherical harmonics. The weights to these basis functions can be calculated in a preprocess step.

Dobashi et al proposed another set of basis functions called the cosine functions, which use less memory than the spherical basis functions and is faster to evaluate. With recent advances in the area of programmable graphics processors, these cosine functions can be evaluated in real time in shaders.

Researchers have constantly looked into using graphics hardware to accelerate modeling of sky and aerial perspective [Dobashi2000][Dobashi2002][Hoffman2002].

Rendering shafts of light through clouds has been done in CPU by numerical integration of in-scattered light along the viewing ray taking into account the visibility of the sample points. The numerical integration can be approximated by a summation of terms along the viewing ray. Dobashi et al achieved this summation by rendering many virtual planes along the viewing ray and accumulating the various terms using blending [Dobashi2000]. The visibility information at any virtual plane is calculated using the standard shadow map technique [Williams1978]. The light information at the virtual plane is evaluated using projective light textures technique. By careful choice of the number of virtual planes and the resolution of the mesh for the virtual plane, photo realistic scenes with atmospheric scattering can be generated at interactive frame rates.

Dobashi et al accelerated their previous techniques of rendering a sequence of virtual or sampling planes and accumulating the terms using blending [Dobashi2002]. The novelty in aerial perspective is that the summation is written as a product of a high frequency term and a low frequency term. The low frequency term can be evaluated accurately in a preprocess step and is written as a product of two terms; one term is stored in the vertices of the mesh grid of the sampling plane and the other is stored as a texture map. A sequence of planes is drawn with blending to get the final image. A rendering from their publication is shown in Figure 7.

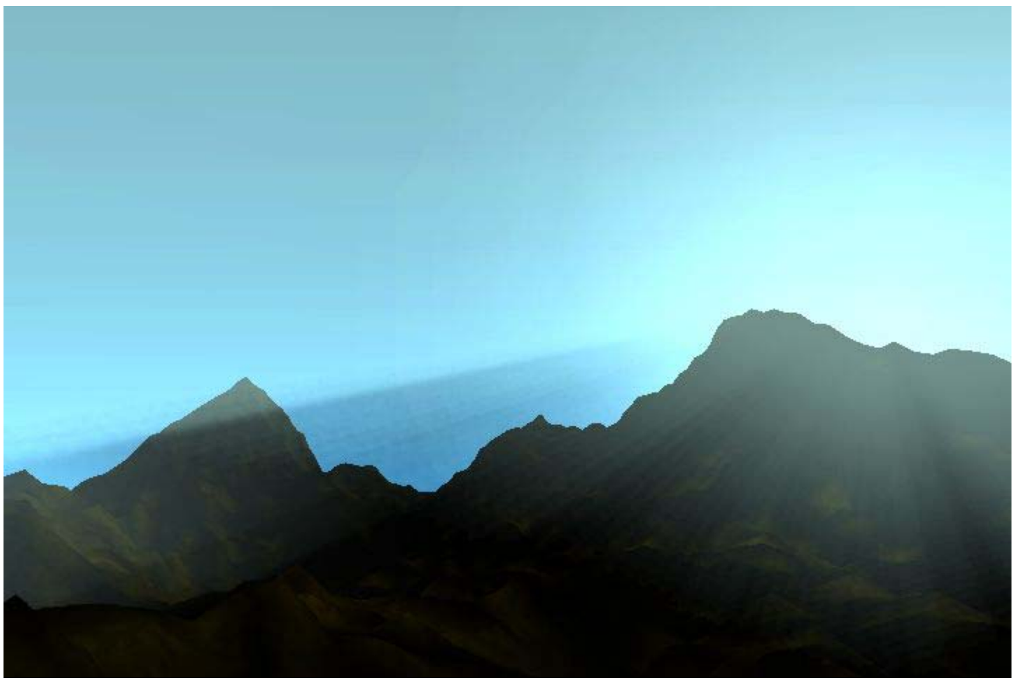

Figure 7: Interactive atmospheric scattering [Dobashi2002].

Modern graphics hardware (e.g. Radeon 8500, Radeon 9700, GeForce 4, GeForce FX) allows the user to specify a vertex shader for transformation and lighting. Hoffman et al presented a new technique to render sky color and aerial perspective in real time using programmable graphics hardware [Hoffman2002]. The two terms for aerial perspective are extinction factor f and in-scattered light L_{in}. These are extended to two kinds of particles - molecules and aerosols. f and L_{in} are calculated in the vertex shader on a per vertex basis and is passed on to the fragment program through the color or texture registers. The final color is computed in the fragment program using the equation

$$L_s = fL_0 + L_{in}\,.$$

With the latest generation of graphics hardware boasting of a full floating point pipeline (e.g. Radeon 9700, Geforce FX), the entire computation of aerial perspective terms (f and L_{in}) can be done at a pixel level rather than a vertex level as was presented. The advantage of a per pixel computation is that it does not require very fine tessellation of the geometry to avoid artifacts due to linear interpolation.

The image in Figure 8 was rendered on 600MHz Pentium III with a Radeon 8500 at about 60 frames per second.

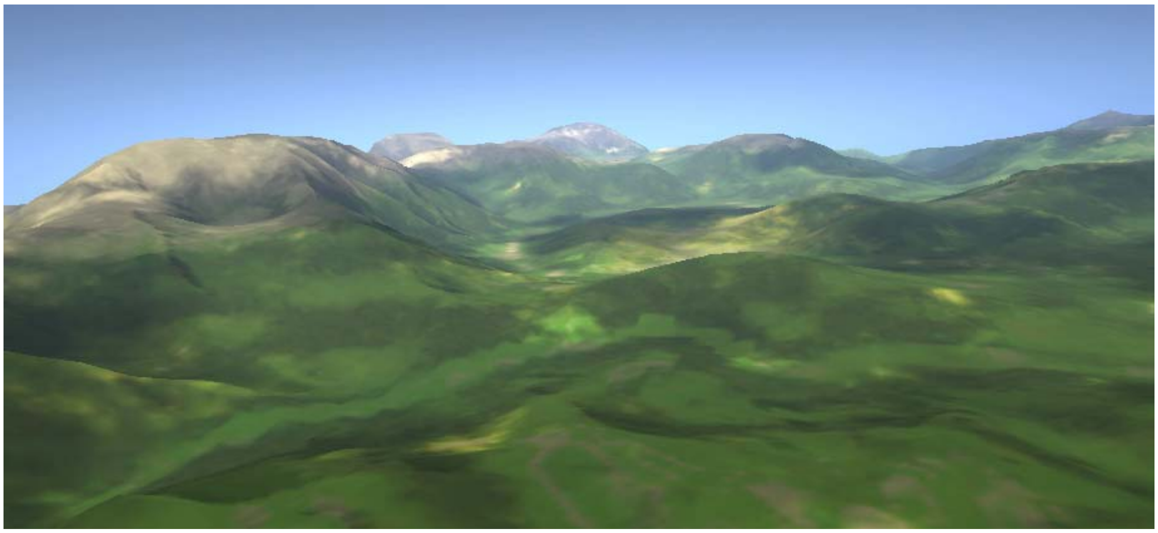

Figure 8: Real time atmospheric scattering [Hoffman2002].

References

[Bullrich1964] Bullrich, K. Scattered radiation in the atmosphere. In *Advances in Geophysics,* 10, Academic Press, 1964.

[CIE1994] CIE-110-1994. Spatial distribution of daylight – luminance distributions of various reference skies. Tech. Rep., International Commission on Illumination, 1994.

[Coulson1975] Coulson, K.L. *Solar and terrestrial radiation.* Academic Press, 1975.

[Dobashi1995] Dobashi, Y., Nishita, T., Kaneda, K., Yamashita, H. Fast display method of sky color using basis functions. In *Pacific Graphics,*1995.

[Dobashi2000] Dobashi, Y., Yamamoto, T., Nishita, T. Interactive rendering method for displaying shafts of light, In *Pacific Graphics,* 2000.

[Dobashi2002] Dobashi, Y., Yamamoto, T., Nishita, T. Interactive rendering of atmospheric scattering effects using graphics hardware. *Proceedings of the conference on Graphics hardware,* 2002.

[Hoffman2002] Hoffman, N., Preetham, A.J. Rendering outdoor light scattering in real time. *Game Developers Conference*, 2002.

[Henyey1941] Henyey, L.G., and Greenstein, J.L. Diffuse reflection in the Galaxy. *Astrophysical Journal 93:70*, 1941.

[Ineichen1994] Ineichen, P., Molineaux, B., and Perez, R. Sky luminance data validation: comparison of seven models with four data banks. *Solar Energy 52*, 4, 337-346, 1994.

[Iqbal1983] Iqbal, M. *An introduction to Solar Radiation*. Academic Press, 1983.

[Kaneda1991] Kaneda, K., Okamoto, T., Nakame, E., and Nishita, T. Photorealistic image synthesis for outdoor scenery under various atmospheric conditions. *The Visual Computer* 7, 5 and 6, 247-258, 1991.

[Klassen1987] Klassen, R.V. Modeling the effect of the atmosphere on light. *ACM Transactions on Graphics* 6, 3, 215-237, 1987.

[Littlefair1994] Littlefair, P.J. A comparison of sky luminance models with measure data from Garston, United Kingdom. *Solar Energy*, 53, 4, 315-322, 1994.

[Lynch1995] Lynch, D.K., and Livingston, W. *Color and light in nature,* Cambridge University Press, 1995.

[McCartney1976] McCartney, E.J. *Optics of the atmosphere*. Wiley publication, 1976.

[Minnaert1954] Minnaert, M. *Light and color in the open air*. Dover, 1954.

[Nishita1996] Nishita, T., Dobashi, Y., Kaneda, K., and Yamashita, H. Display method of the sky color taking into account multiple scattering. In *Pacific Graphics*, pp. 117-132, 1996.

[Preetham1999] Preetham, A.J., Shirley, P., Smits, B.E. A practical analytical model for daylight. In *Proceedings of Siggraph*, 91-100, 1999.

[Robinson1966] Robinson, N. *Solar radiation*. Elsevier Publishing Company, 1966.

[Rayleigh1871] Rayleigh, L. On the scattering of light by small particles. *Philosophical Magazine 41*, 447-451, 1871.

[VanDeHulst1981] van De Hulst, H.C. Light scattering by small particles. *Dover Publications*, 1981.

[Williams1978] Williams, L. Casting curved shadows on curved surfaces. In *Proceedings of Siggraph*, 270-274, 1978.

[Wyszecki1982] Wyszecki, G., and Stiles, W.S. *Color Science*. Wiley-Interscience publication, 1982.